A Note on Stability of a damped wave coupled with thermal effect

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Abstract :

We consider the vibrations of a beam with Kelvin-Voigt damping coupled with an expectedly dissipative effect through heat conduction, which is clamped at one end and free at other end. We are obtaining the uniform exponential stability of the vibrations using multiplier techniques.

Keywords : Energy decay estimate, Exponential stability, Kelvin-Voigt damping.

1. Introduction :

In this paper, we consider the vibrations of a cantilever beam of length L, which is clamped at one end and free at other end. It is made of a viscoelastic material with Kelvin-Voigt type damping. Consequently the vibrations of the beam satisfied the linear partial differential equation (cf. Liu et. al. [8])

$$u_{tt} = c^2 u_{xx} + 2\delta u_{xxt} \quad \text{in } (0, L) \times \mathbb{R}^+, \tag{1.1}$$

where $u = u(x,t)$ and $\mathbb{R}^+ := (0, \infty)$. The constants c, δ denotes respectively coefficient of wave velocity and coefficient of viscous damping for such beam.

In last few decades the use of flexible structures is on rise. The theory of stabilization of flexible structural system has been a topic of interest in view of vibration control of various structural elements. The vibrations of flexible structures are usually non-linear in practice. As the non-linear study of such structures is rather cumbersome for analytical treatment, so linearized mathematical models are chosen for simplicity and concise results. The linearized vibrations of flexible structures are usually governed by partial differential equations, particularly, second order wave equation and fourth order Euler-Bernoulli beam equation.